3.1: Systems of Two Equations in Two Unknowns

Definition 1. A <u>linear equation in two unknowns</u> is an equation that can be written in the form

$$ax + by = c$$

with a, b and c being real numbers. The number a is called the <u>coefficient of x</u> and b is called the <u>coefficient of y</u>. A <u>solution</u> of an equation consists of a pair of numbers for x and y that satisfy the equation.

Example 1. In the linear equation $3x - y = 15$, the coefficients are $a = 3$ and $b = -1$. The point $(x, y) = (5, 0)$ is a solution, because $3(5) - (0) = 15$. The graph represents all solutions to the equation.

Example 2. (Solving a system of two equations graphically and algebraically) Find all solutions (x, y) of the following system of two equations:

$$x + y = 3 \quad x - y = 1.$$

Example 3. Solve the system both graphically and algebraically and verify that you get the same example.

$$3x + 5y = 0 \quad 2x + 7y = 1.$$

Example 4. Solve the system

$$x - 3y = 5 \quad -2x + 6y = 8.$$

Example 5. Solve the system

$$x + y = 2 \quad 2x + 2y = 4.$$

Example 6. (Blending) Acme Baby Foods mixes two strengths of apple juice. One quart of Beginner's juice is made from 30 fluid ounces of water and 2 fluid ounces of apple juice concentrate. One quart of Advanced juice is made from 20 fluid ounces of water and 12 fluid ounces of apple juice concentrate. Every day Acme has available 30,000 fluid ounces of water and 3,600 fluid ounces of concentrate. If the company wants to use all the water and concentrate, how many quarts of each type of juice should be mixed?

Example 7. A medieval alchemist's love potion calls for a number of eyes of newt and toes of frog, the total being 20, but with twice as many newt eyes as frog toes. How many of each is required?

---

8. (Equilibrium Price) The demand for refrigerators in West Podunk is given by

$$q = -\frac{p}{10} + 100$$

where q is the number of refrigerators that the citizens will buy each year if they are priced at p dollars each. The supply is given by

$$q = \frac{p}{20} + 25$$

where q is the number of refrigerators the manufacturers will be willing to ship into town each year is they are priced at p dollars each. Find the equilibrium price and the number of refrigerators that will be sold at that price.